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# Modeling and simulation of machine tool dynamics

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**Abstract.** A method of establishing the dynamic analytic model of machine tool considering joint dynamic characteristics is developed and analyzed. The method is based on study of joint dynamic characteristics to obtain dynamic values of combined surface incorporating joint dynamic data, and build up the dynamic model using design drawing or actual structure. Firstly, according to the structure characteristics of the machine tool and the vibration displacement for each component in exciting test, the motion coordinates of each component are selected, and the machine tool is simplified reasonably. Secondly, analyzing and studying the specific joint, according to the way and condition of the joint, the equivalent dynamic parameters of each joint are calculated by applying the general joint surface dynamic data. Calculation methods of dynamic values for some typical joints in machine tool structures are analyzed and presented. Based on the second type of Lagrange equations, the dynamic model can be finally obtained. As a practical example, a universal tool milling machine which is manufactured by Kunming Milling Machine Plant is represented by a dynamic model with 21 degrees of freedom, the computations of dynamic characteristics and response are completed and the results agree with that of exciting test. The dynamic model can well simulate the actual dynamic characteristics, and thus proving the method effective and applicable.

**Key words.** Machine tool, modeling, dynamic model,dynamic performance, joints,modal flexibility.

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## 1. Introduction

In order to analyze the structural dynamics of machine tool and realize dynamically optimum design, it is necessary to build up dynamic analytic model that can be simulated the machine tool structures. For complex machine tool structure system, it is still very difficult to establish the dynamic model of its physical coordinates. So far, the methods of building up the dynamic analytic model can be divided up three kinds as a general rule:

The first method, by applying machine tool drawing or a real structure, simplify machine tool to a certain extent in accordance with different ways, and build up a lumped-parameter model, a beam distributed model or a finite element model correspondingly[1-3]. Because of the complexity of machine tool structures, especially joints, this kind of modeling now is rather limited[4], and the modeling precision is below to dynamically optimum design requirements.

In second method, based on the theory of experimental modal analysis, using experimental data of modal test for machine tool, the modal model is established [5-7]. First, by modal test data, the modal parameters are obtained, then, according to the evaluated coupling distributions of mass, stiffness and damping of machine tool, initial geometrical state of the model can be given, and the place and direction of motion coordinates are determined. The dynamic structural parameters are finally identified in accordance with transformation of modal coordinates and physical coordinates. This kind of modeling was limited be the modal numbers excited, so that the dynamic model with the few degrees of freedom can only be obtained.

The third method is substructure method [8], which divided the total structure into many parts; each part is called a substructure. In the light of the structural characteristics of each substructure, a lumped-parameter model, a beam distributed model or a finite element model can be modeled correspondingly. The dynamic characteristics of substructure are determined by analysis or experiment. Then by applying modal synthesis method or mechanical impedance method, the dynamic model of complete structure is obtained by synthesis all substructures according to joints conditions [9-10].

With developing the experimental modal analysis technology, although it is easy to determine the modal mass modal stiffness and modal damping ration for each modal of machine tool, as viewed from design improvement and optimum design, the obtained modal model could not related directly to the specific design improvement and optimum design, only some approximately concussions can be obtained. On the contrary, the dynamic model in physical coordinates is convenient and direct in point of this.

Based on the first method as stated above, a method of modeling the dynamic analytic model of machine tool considering joint dynamic characteristics and using design drawing is developed. Above all, in accordance with the characteristics of structure and displacement in exciting test for each component of the machine tool, the motion coordinates for each component are decided, and the machine tool is simplified reasonably. Upon that, making a study of the specific joint, the equivalent parameters of each joint can be calculated by applying the general joint surface dynamic data. Based on the second type of Lagrange equations, the dynamic equations of the mechanical model system can be finally developed.

## 2. The Dynamic Analytic Model for the Machine Tool

This universal tool milling machine is a middle-sized product with broad versatility. In accordance with the structural features and distributions of this machine, it is divided up eight parts: (1) Bed (including bottom base); (2) Horizontal spindle body (including main motor, motor stand and vertical spindle body); (3)Crossbeam; (4) Hanger; (5) Horizontal cutter arbor; (6) Horizontal table; (7) Vertical table; (8) Compound slide.

Compound slide fits bed with rectangular slideway. Between horizontal spindle body and bed, crossbeam and horizontal spindle body, hanger and crossbeam as well as vertical table and compound slide are all fitted each other with dovetail slideway. Horizontal table and vertical table are joined by four bolts.

In accordance with the characteristics of displacement in exciting test of the machine tool, the machine is represented by a dynamic model involving lumped and distributed masses with 21 degrees of freedom. By applying the second type of Lagrange equations, the differential equations of machine tool motion are obtained as follows

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_j}\right]-\frac{\partial T}{\partial q_j}+\frac{\partial U}{\partial q_j}+\frac{\partial D}{\partial \dot{q}_j}=Q_j \qquad j=1,2,\cdots,n \qquad n=21 \tag{1}$$

Where T is the total kinetic energy of the system, $q_j$ is the generalized coordinate of the system, U is the total potential energy of the system, D is the Rayleigh energy dissipation function, and $Q_j$ is the exciting force. According to the dynamic model, the total kinetic energy T, the total potential energy U and Rayleigh energy function D of the structure are calculated, substitute T, D, U in equation (1), then we obtain

$$\mathbf{M}_{21\times 21}\ddot{\mathbf{q}}_{21\times 1}+\mathbf{C}_{21\times 21}\dot{\mathbf{q}}_{21\times 1}+\mathbf{K}_{21\times 21}\mathbf{q}_{21\times 1}=\mathbf{F}(t)_{21\times 1} \tag{2}$$

Masses of each component involved in the model are decided by weighing actual structures. In the light of the principle of compound pendulum, moments of inertia are determined by measuring the oscillation frequency in gravity field, while neglecting the effect of friction moment on the edge of the blade bearing.

The key to modeling is to determine the physical parameters of joints in the model accurate comparatively. Basing on analysis of characteristics of joints, by applying the general joint surface dynamic data $k_i(P_n)$ and $c_i(P_n)$ ($i$=1, 2)[11], the equivalent spring stiffness and viscous damping coefficients of each joint in the model are calculated in this paper. $k_i(P_n)$ and $c_i(P_n)$ ($i$=1,2), are equivalent spring stiffness and damping coefficient values per unit area, which are obtained by experiment and computer simulation, they are functions of contact pressure. Subscripts $i$=1,2 represent shear direction and normal direction respectively. It is confirm that the joint dynamics data $k_i(P_n)$ and $c_i(P_n)$ ($i$=1,2), which depend on the mean contact pressure, can be applied satisfactorily to general joints which have the same contact surface properties but differ in shape and contact area.

In the joints of machine tool structure, which with fixed and sliding of joint ways, as well as with surface contact, rectangular slideway and dovetail slideway of joint patterns, and with different the state of forces, therefore, it is necessary to further analyze for calculating the parameters of these joints by using general joint surface dynamic data.

For a contact surface, the forms of dynamic force supported on contact surface are determined by modal shapes. The contact surface can be supported six different forms of dynamic forces, which are generalized forces over six coordinates as shown in Figure 1. These dynamic forces are normal force $F_y$ along axis y, shearing force $F_x, F_z$ along axis x and $z$, bending moment $M_{\theta x}, M_{\theta z}$ around axis x and $z$, shear bending moment $M_{\theta y}$ around axis y. The contact surface may be supported all of them or any of these forces. Integral over area replaced by the point G, the equivalent springstiffness and damping coefficients at point G are obtained as indicated in the following equations:

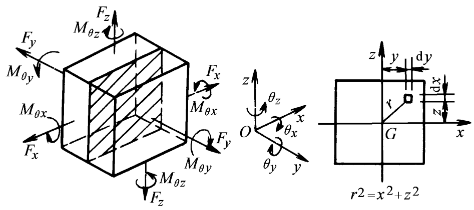


Fig. 1. Diagrammatic sketch of the forces on contact surface

$$\left.\begin{array}{l}K_x = \iint k_1(P_n)dxdz = K_z \\ K_y = \iint k_2(P_n)dxdz \\ K_{\theta y} = \iint (y^2+z^2)k_1(P_n)dxdz \\ K_{\theta x} = \iint z^2 k_2(P_n)dxdz \\ K_{\theta z} = \iint y^2 k_2(P_n)dxdz\end{array}\right\} \quad (3)$$

$$\left.\begin{array}{l}C_x = \iint c_1(P_n)dxdz = c_z \\ C_y = \iint c_2(P_n)dxdz \\ C_{\theta y} = \iint (y^2+z^2)c_1(P_n)dxdz \\ C_{\theta x} = \iint z^2 c_2(P_n)dxdz \\ C_{\theta z} = \iint y^2 c_2(P_n)dxdz\end{array}\right\} \quad (4)$$

Based on the equation (3) and (4), for dovetail slideway joint, shown in Figure 2, the equivalent spring stiffness at point G can be derived by calculating the spring stiffness around point G as follows

$$K_x = 2(K_{Dx} + K_{Hx} + K_{Ex}\cos\theta + K_{Ey}\sin\theta)$$

$$K_y = 2(K_{Dy} + K_{Hy} + K_{Ex}\sin\theta + K_{Ey}\cos\theta)$$

$$K_z = 2[K_{Dz} + K_{Hz} + K_{Ez})$$

$$K_{\theta x} = 2[K_{D\theta x} + K_{H\theta x} + K_{E\theta x}\cos\theta + K_{E\theta y}\sin\theta + \frac{x_3^2}{4}(K_{Hz} + K_{Dz})]$$

$$K_{\theta y} = 2[K_{D\theta y} + K_{H\theta y} + K_{E\theta x}\sin\theta + K_{E\theta y}\cos\theta +$$

$$\frac{(x_1+x_2)^2}{16}K_{Hz} + \frac{(x_5+x_6)^2}{16}K_{Dz} + \frac{(x_6+x_2)^2}{16}K_{Ez}]$$

$$K_{\theta z} = 2[K_{D\theta z} + K_{H\theta z} + \frac{(x_1+x_2)^2}{16}K_{Hy} + \frac{(x_5+x_6)^2}{16}K_{Dy} + K_{E\theta z} +$$

$$\frac{(x_6+x_2)^2}{16}(K_{Ex}\sin\theta + K_{Ey}\cos\theta) + \frac{x_3^2}{4}(K_{Hx} + K_{Dx})]$$

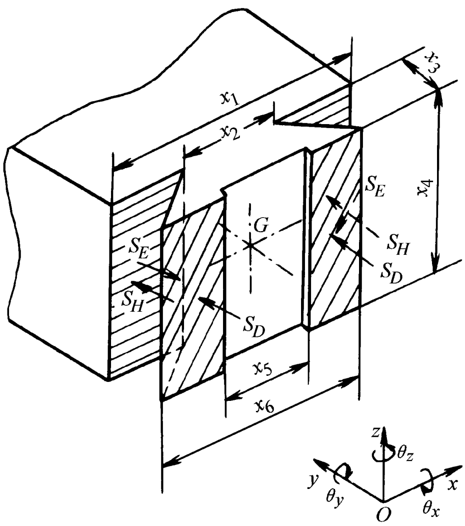


Fig. 2. Dovetail slideway joint

For rectangular slideway joint, shown in Figure 3, the equivalent spring stiffness at point G can be derived as follows

$$K_x = 2(K_{Dx} + K_{Hx} + K_{Ey} + K_{Ty})$$

$$K_y = 2(K_{Dy} + K_{Hy} + K_{Ex} + K_{Tx})$$

$$K_z = 2[K_{Dz} + K_{Hz} + K_{Ez} + K_{Tz})$$

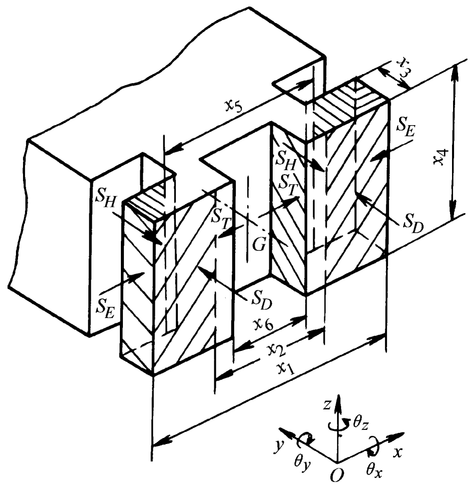


Fig. 3. Rectangular slideway joint

$$K_{\theta x} = 2[K_{D\theta x} + K_{H\theta x} + K_{E\theta y} + K_{T\theta y} + \frac{x_3^2}{4}(K_{Hz} + K_{Dz})]$$

$$K_{\theta y} = 2[K_{D\theta y} + K_{H\theta y} + K_{E\theta x} + K_{T\theta x} +$$

$$\frac{(x_1 + x_5)^2}{16}K_{Hz} + \frac{(x_1 + x_2)^2}{16}K_{Dz} + \frac{x_6^2}{4}K_{Tz} + \frac{x_1^2}{4}K_{Ez}]$$

$$K_{\theta z} = 2[K_{D\theta z} + K_{H\theta z} + K_{E\theta z} + K_{T\theta z} + \frac{(x_1 + x_5)^2}{16}K_{Hy} +$$

$$\frac{(x_1 + x_2)^2}{16}K_{Dy} + \frac{x_3^2}{4}(K_{Hx} + K_{Dx}) + \frac{x_6^2}{4}K_{Tx} + \frac{x_1^2}{4}K_{Ex}]$$

Similarly, the equations for calculating the equivalent damping coefficients are derived. In the first subscripts D, H, E, T correspond to the joint surface $S_D$,$S_H$, $S_E$ and $S_T$, while the second subscripts $x, y, z, \theta_x, \theta_y, \theta_z$ correspond to the directions of six coordinates respectively.

By applying above equations, the equivalent spring stiffness and damping coefficients in the model are calculated, and the results for equivalent spring stiffness of joins in the model are exhibited in Table 1.

Table 1. The values of equivalent stiffness of each joint

| Joint | Stiffness（N/m; N·m/rad） | | Joint | Stiffness（N/m; N·m/rad） | |
|---|---|---|---|---|---|
| Joint of bed and horizontal spindle body | $K_{x}^{\text{①}}$ | $7.642\times10^{8}$ | Crossbeam and hanger | $K_{\theta x}^{\text{③}}$ | $7.366\times10^{6}$ |
| | $K_{\theta x}^{\text{①}}$ | $3.629\times10^{7}$ | | $K_{\theta z}^{\text{③}}$ | $1.164\times10^{7}$ |
| | $K_{\theta z}^{\text{①}}$ | $5.473\times10^{7}$ | Horizontal table and vertical table | $K_{x}^{\text{⑥}}$ | $2.512\times10^{9}$ |
| Bed and compound slide | $K_{x}^{\text{⑧}}$ | $3.829\times10^{8}$ | | $K_{z}^{\text{⑥}}$ | $2.512\times10^{9}$ |
| Horizontal spindle body and crossbeam | $K_{x}^{\text{②}}$ | $2.012\times10^{8}$ | Vertical table and compound slide | $K_{x}^{\text{⑦}}$ | $7.218\times10^{5}$ |
| | $K_{\theta x}^{\text{②}}$ | $6.725\times10^{6}$ | | $K_{\theta x}^{\text{⑦}}$ | $4.538\times10^{7}$ |
| | $K_{\theta z}^{\text{②}}$ | $1.054\times10^{7}$ | | $K_{\theta v}^{\text{⑦}}$ | $4.341\times10^{7}$ |

Note: In the table the unit of line stiffness is N/m; and angular stiffness is N·m/rad.

## 3. Dynamic Characteristics Analysis and Computation

After the motion equation of the structural system is obtained, the characteristic equation can be written as follows

$$\mathbf{K}\phi_i = \omega_i^2\mathbf{M}\phi_i \tag{5}$$

Where $\phi_i$ is r-order eigenvector, the natural frequencies $\omega_{n1}, \omega_{n2}, \cdots, \omega_{n21}$ and vibration vectors $A^1, A^2, \ldots, A^{21}$ of the structural system can be obtained respectively by solving above equation.

The actual cutting conditions are simulated approximately by the relative exciting shown in the Figure 4. In the case of relative exciting between cutter and workpiece, a pair of equal and opposite force $-F_e e^{i\omega t}$ at the point of horizontal cutter arbor and $F_e e^{i\omega t}$ at the point of horizontal table are applied along the direction of $\alpha$ angle, $-F_e e^{i\omega t}$ and $F_e e^{i\omega t}$ are decomposed into a pair of orthogonal forces along the axis x and z respectively. Thus, the exciting force column matrix can be expressed as

$$\mathbf{F}(t) = (00000000000, -F_{e12x}e^{i\omega t}, F_{e13z}e^{i\omega t}, 00, F_{e16x}e^{i\omega t}, -F_{e17z}e^{i\omega t}, 0000)^T = \mathbf{F}e^{i\omega t} \tag{6}$$

In accordance with theory of modal analysis, the cutter-workpiece relative displacement in the normal direction of machined surface under relative of exciting can

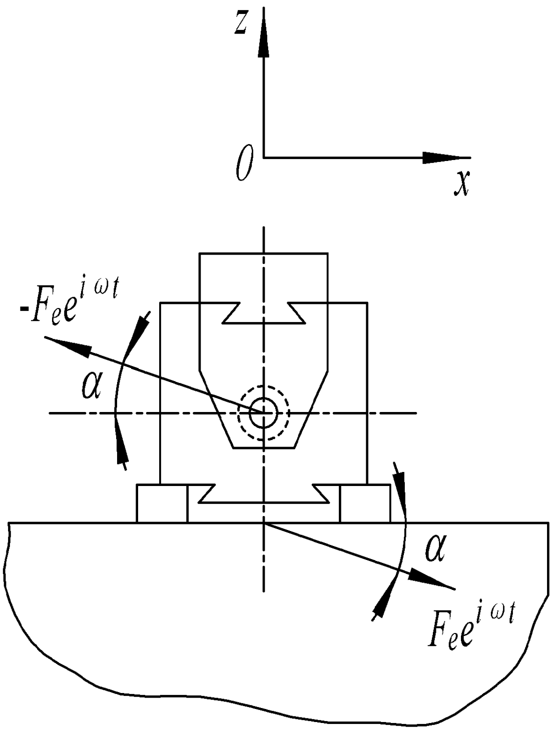


Fig. 4. Diagram of exciting force

be derived as

$$X_c = \sum_{r=1}^{21} \frac{\left[\left(A_{16}^{(r)} - A_{12}^{(r)}\right)\cos\alpha + \left(A_{13}^{(r)} - A_{17}^{(r)}\right)\sin\alpha\right] F_e \left(A_{13}^{(r)} - A_{17}^{(r)}\right)}{K_r\left[1 - \left(\frac{\omega}{\omega_{nr}}\right)^2 + i\cdot 2\xi_r\left(\frac{\omega}{\omega_{nr}}\right)\right]} \qquad (7)$$

Where, $A_{12}^{(r)}$, $A_{13}^{(r)}$, $A_{16}^{(r)}$, $A_{17}^{(r)}$ — the r-order modal vectors corresponding to the generalized coordinate $q_{12}$ ,$q_{13}$, $q_{16}$, $q_{17}$; $K_r$—the r-order modal stiffness; $\xi_r$ — the r-order modal damping ratio; $\omega$, $\omega_{nr}$— exciting frequency and the r-order natural frequency; $\alpha$ — the included angle between exciting force $F_e e^{i\omega t}$ and horizontal direction.

Thus, the compliance of cutter-workpiece at the cutting point can be written as follows

$$W_c = \frac{X_c}{F_e} = \sum_{r=1}^{21} \frac{\left[\left(A_{16}^{(r)} - A_{12}^{(r)}\right)\cos\alpha + \left(A_{13}^{(r)} - A_{17}^{(r)}\right)\sin\alpha\right] \left(A_{13}^{(r)} - A_{17}^{(r)}\right)}{K_r\left[1 - \left(\frac{\omega}{\omega_{nr}}\right)^2 + i\cdot 2\xi_r\left(\frac{\omega}{\omega_{nr}}\right)\right]} \qquad (8)$$

In this equation, if let $\omega = 0$, the relative static compliance $(f_{ce})_s$ and modal flexibility $(f_{ce})_r$ of machine tool structure in cutting force direction and in the normal direction of machined surface at the cutting point is found to be

$$(f_{ce})_s = (\frac{X_c}{F_e})_{w=0} = \sum_{r=1}^{21} \frac{\left[\left(A_{16}^{(r)} - A_{12}^{(r)}\right)\cos\alpha + \left(A_{13}^{(r)} - A_{17}^{(r)}\right)\sin\alpha\right]\left(A_{13}^{(r)} - A_{17}^{(r)}\right)}{K_r} \tag{9}$$

$$(f_{ce})_r = \frac{\left[\left(A_{16}^{(r)} - A_{12}^{(r)}\right)\cos\alpha + \left(A_{13}^{(r)} - A_{17}^{(r)}\right)\sin\alpha\right]\left(A_{13}^{(r)} - A_{17}^{(r)}\right)}{K_r} \tag{10}$$

Or

$$\sum_{r=1}^{21} \frac{(f_{ce})_r}{(f_{ce})_s} = \frac{(f_{ce})_1}{(f_{ce})_s} + \frac{(f_{ce})_2}{(f_{ce})_s} + \cdots + \frac{(f_{ce})_{21}}{(f_{ce})_s} = 1.0 \tag{11}$$

The modal damping ratio $\xi_r$ used in calculation is obtained by analyzing the machine tool relative pseudo-random exciting test data processing by analyzer 7T17S. Figure 5 shows the vibration modal shape of the main modes of 152.7Hz and 187.7Hz. Figure 6 compares the computed frequency response at cutting point with that of exciting test. The dotted lines indicate the computed results, while the solid line is the relative harmonic exciting results. Table 2 compares the modal frequencies in relative pseudo-random exciting with computed natural frequencies. The correspondence between the calculated results and the measured results indicates that the dynamic analytic model established is in line with the experimental situation, and the model can simulate the dynamic characteristics of the actual structure well.

Table 2. Comparison between natural frequencies and modal frequencies

| Modal order | Computed results | Experimental value | | Modal order | Computed results | Experimental value | |
|---|---|---|---|---|---|---|---|
| | Natural frequency(Hz) | Modal frequency(Hz) | Modal damping ratio | | Natural frequency(Hz) | Modal frequency(Hz) | Modal damping ratio |
| 1 | 17.586 | 17.678 | 0.139 | 9 | 301.210 | 300.094 | 0.0250 |
| 2 | 39.154 | —— | —— | 10 | 365.411 | 372.533 | 0.0134 |
| 3 | 64.301 | 65.048 | 0.0384 | 11 | 410.936 | 410.122 | 0.0244 |
| 4 | 109.278 | 123.908 | 0.0503 | 12 | 460.676 | —— | —— |
| 5 | 156.286 | 151.379 | 0.0412 | 13 | 470.350 | —— | —— |
| 6 | 192.470 | 187.650 | 0.0399 | 14 | 491.494 | 491.328 | 0.0178 |
| 7 | 266.715 | —— | —— | 15 | 526.796 | 533.787 | 0.0117 |
| 8 | 279.518 | —— | —— | 16 | 1146.472 | 990.003 | 0.00253 |

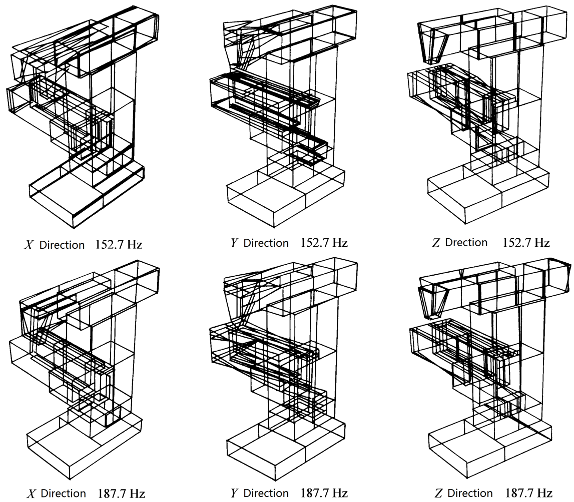


Fig. 5. Vibration modal shape of machine tool

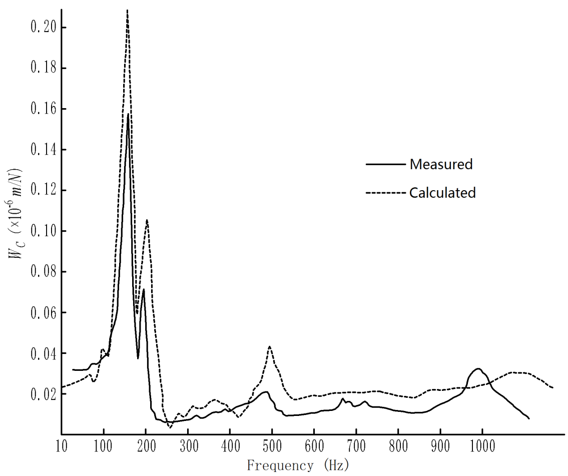


Fig. 6. Computed and measured results of the relative compliance

## 4. Conclusions

1. The method to build the dynamic model of machine tool structure considering joint dynamic characteristics is feasible and applicable to engineering.

2. The dynamic analytic model established simulates the dynamic characteristics of the actual structure well, the computed results agree with that of exciting test.

3. With Lagrange equations based on principle of energy to establish the equation of motion of machine tool structure, so it can be easily calculated energy distribution of each component in the system, and the energy ratio of a part in the system can be computed consequently. Based on this, dynamic optimum design and structural

improvement can be carried on, which will discuss in other paper.

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